Gradus Vol 3, No 1 (2016) 134 - 141
ISSN 2064-8014

RESEARCH BASED EXPERIENCE IN THE DEVELOPMENT OF EARLY NUMBER SENSE IN EARLY CHILDHOOD CLASSROOMS IN NEW YORK CITY

Judit Kerekes[1*]
[1]Mathematics Education, CUNY Staten Island, Staten Island, NY, USA

Keywords:
Mathematics Education
Number Sense
Early Childhood Education
Auto-Ethnography

Article history:
Received 15 Oct. 2015
Revised 15 Feb. 2016
Accepted 28 Feb. 2016

Abstract
A native Hungarian professor has taught in New York City for 20+ years. This paper presents an auto-ethnographic study of a her work with early childhood teachers in pre-service and in-service settings.The constructivist perspective provides avantage point to understand sucessful development of early childhood math education strategies.

1 Introduction

This paper presents an auto-ethnographic study of my work with early childhood pre-service and in-service teachers in New York City. I had the opportunity to start my teaching career in my native Hungary before moving to the United States, where I have been a professor for over 20 years. During this time, I have developed and refined many strategies to bring constructivist practices into early childhood math education.

Auto-ethnographies provide a powerful means to document and analyze life experience (Ellis, 2009) [1]. Such research may synthesize theory with data and narrative. The foundational theoretical framework for this study is constructivist teaching and learning methods.

1.1 Background

Science, Technology, Engineering and Mathematics (STEM) professions continue to be in great demand worldwide, and yet educators find it difficult to cultivate learners' interests in these fields. Part of the solution may lie with how we teach math concepts and understanding during the early years of formal education. This paper documents my experience and research in building positive mathematics learning experiences among young students. Specifically, I have invested in helping teachers learn how to use
effective, enjoyable, and real-life activities. These learning strategies provide an innovative, research-based, and field-tested approach to teaching math concepts to young children. An example of this is, working with
young children to develop number sense, and then progressing to experiences with part-whole concepts.

This paper is based upon an extended action research (Hinchey, 2008) [2], which has revealed how using a Rekenrek manipulative, (Tournaki, Bae, & Kerekes, 2008) [3], play techniques (Streeflund, 1991) [4]
and Think Aloud strategies (van Someren, Barnard, & Sandberg, 1994) [5], provide foundational experiential
learning for young mathematicians. Specifically, during these early years, the students engaged

*Corresponding author. Tel.: +01 718 982 4112; fax: +01 718 982 3743
E-mail address. judit.kerekes@csi.cuny.edu

in real-life scenarios to experience and discover number sense, arithmetic skills, and internalize essential pre-Algebra concepts. An auto-ethnographic study in this action research is the focus of this paper's analysis.

An essential shift in such classrooms is critical for such learning. In traditional classrooms, young students study information based on the authority of the teacher. In these constructivist Math classes, young mathematicians are energized through their personal discovery of math concepts. They play with numbers, use small figures and toys to work out real life word problems and experience the excitement of numbers, number relationships, number sense, and part-whole concepts.

2 Research Method

The method of auto-ethnography provides a way to use ethnography to conduct qualitative research. The researcher's perspective is used in the auto-ethnographic method in order to examine cultural assumptions within the context of his/her lived experience (Chang, 2008; Reed-Danahey, 1997)[6], [7].

In the field of anthropology, auto-ethnography was developed as an alternative ethnographic method. Subsequently, auto-ethnography's use extended to include research in other areas such as the social sciences, for instance, communications, sociology, and education (Ellis, 2009)[1]. Teacher education has recognized auto-ethnography as an especially valuable research method and lens for scaffolding critical reflection and reflective practice (Taylor, Klein, & Abrams, 2014)[8]. From a philosophical perspective, researchers engaged in auto-ethnography recognize that they, themselves, are individual "texts" because as individuals they each have specific epistemologies, values and beliefs (Richardson, 1997 as cited in Ellis, 2009)[1]. Furthermore, the literature refers to research dialogue as a collective, or coauthored, "text."

For this research study, the information was documented through guided journal writing, document review (lessons, class activities, class notes, review of previous articles and research papers), and recorded informal discussions with colleagues (Chang, 2008; Izzo, 2006)[6],[9]. It was especially possible to use this method because I keep extensive records (1992-2015) of my teaching materials, student work and presentations related to math education teaching methods. In my reflections and journaling, these documents reminded me not only of my instructional strategies, but also my thoughts about benefits, difficulties, struggles, and victories while using them. Furthermore, the accounts brought to mind my graduates and their work in math education and elementary education today.

2.1 Role of the Researcher-Participant

Researchers are the central instrument in qualitative research. In the opening section of this article, I briefly described my background and perspective. However it is essential to also describe my perspective on mathematics education and teaching philosophy in general.

As one can recognize from the prior section, I am a constructivist and humanist in my educational philosophy. I believe the role of the educator is to cultivate the greatest human potential possible from within our students. Furthermore, I recognize that teaching math education and mathematics is one of the most wonderful things one could engage in as a career. I love and value mathematics inherently and I am thrilled by the excitement and joy of sparking math knowledge, understanding, and enthusiasm in my students, among young mathematicians and among my teacher candidates alike. It is from this perspective, and because of these values, that I have engaged in constructivist, realistic math learning (Fruedenthal, 1987) [10] and been innovative throughout my career. Moreover, this research about constructivist, realistic math learning has been conducted from an orientation within the worldview and practice of constructivism.

2.2 Analysis

The qualitative data were coded using thematic, open axial coding (Creswell, 2009; Merriam, 2009) [11], [12]. This approach identified the themes among the data sources by iteratively examining them for themes and patterns. The themes and patterns are shared within the section of Findings and Discussion. In addition, in order to specifically document the nature and details of the approaches described in the article, this articles includes two examples of Real-life Applications for Early Childhood Math Teaching and Learning.

3 Findings and Discussion

Critical elements of discussion of the data include a brief overview of relevant constructivist theory and research, samples of real-life applications for early childhood math teaching and learning, and a brief overview of current trends in USA early childhood mathematics education (i.e., Common Core State Standards). [14]

3.1 Constructivist Literature and Math Learning

The constructivist literature extends many years of educational theory, research and practice in experiential learning which facilitates student learning by discovery. This section provides a specific overview of the literature related to constructivism research in the area of math education.

In Fosnot and Dolk's 2001 [15] definition of a mathematician, they describe the essential link between real-life context and classroom instruction. In our pre-service and in-service teacher programs and the classrooms of our students, this link has become a powerful focus for teaching mathematics through constructivist methods. They said, "Being a mathematician means thinking mathematics outside the classroom as well as in it. It means being willing to work on problems at home, to wonder about them during your commute to work, to raise your own mathematical inquiries." (Fosnot & Dolk, 2001, p. 179) [15].

Regarding the theoretical roots of realistic mathematics education, Freudenthal is widely recognized as the founder. Treffers (1991 in Streefland, 1991)[4] describes this historic transition from traditional math education to realistic:

> "He [Freudenthal] was the one to put Wiskobas on the right track: away from formalistic New Math, directed a reality. His didactical realism is coloured by idealism. His ideas emphasize rich thematic contexts, integration of mathematics with other subjects and areas of reality, differentiation within individual learning processes and the importance of working together in heterogeneous groups." (Treffers in Streefland, 1991, p. 19). [4]

It was Van den Brink's research which demonstrated the concrete benefits which realistic mathematics education practice could have on basic math skills (such as addition and subtraction). His research included the study of children's playing and creativity as learning. In these studies, they used miniature toy buses, bus stops and toy passengers. The children played with the materials and then a facilitator/teacher guided them through scenarios where the miniature passengers (people or animals), boarded and disembarked from the toy bus at various points in its route. Findings from this research indicated that the young learners (pupils) learning with these techniques gained greater insight, and more quickly, than those who were in the control group using traditional mathematics instruction (Van den Brink in Streefland, 1991) [4].

Math education literature clearly states as fact that materials do not transmit knowledge. For example, popular, traditional explanation of realistic instruction theory stated:

"Material is only an aid to solve certain practical problems in a certain context. Their understanding and insight are supported by the context, which can serve as a situation model. In the 'informal solution' variant the material is used to elicit (mental) arithmetic actions which other children have previously developed themselves." (Gravemeijer in Streefland, 1991, pp. 75-76). [4]

Moreover, Van den Brink's 1989 research (referenced in Streefland (1991)) [4], further revealed that the young children learned more quickly, with greater understanding, while also internalizing their own understandings of addition and subtraction when incorporating play and creativity. Specifically, this research demonstrated that, within this group, 50% fewer practice exercises were needed to achieve proficiency in arithmetic skills. Further, the use of mathematics learning strategies such as arrow language was effective in building abstract understanding through application. A major characteristic of the learning activities which incorporate play and creativity is that they are also integrated with real-life scenarios, concepts and connections. By design, these young learners are learning arithmetic in settings which are relevant to their daily experiences, instead of mathematics being restricted to isolated, repetitive activities.

In 1987, Freudenthal had discussed the benefits of real-life contexts for math learning when he described the process of students moving "back and forth between the real world and the world of symbols" (p. 83) [10]. Freudenthal described how instead of stumbling over the critical stage of comprehending mathematics symbols, to these students, the mathematical symbols are not abstract, because they have developed their own connections between the mathematical concepts/operations and real life situations.

A further innovation embedded in constructivist, real-life context math instruction is that testing no longer has to restrict creativity. Instead, because learners are engaged in creative thinking, building multiple solutions, and being able to determine why arithmetic principles work, students routinely engage in building clever strategies. In the process, their instructors have a much greater scope for assessment of the students' capabilities and can guide them to more advanced instructional experiences. Van den Heuvel-Panhuizen and Gravemeijer (In Streeflund, 1991) stated, "This [format] involves a reversal of the part played by tests in innovation. Rather than thwart innovations, tests would contribute towards improving education." (p. 154) [4].

3.2 Constructivist Literature and Teaching Math

In order to incorporate these innovative approaches to math education, it is essential to use different methods of teacher education. For instance, when teachers don't like math themselves (e.g., peer pressure, lack of training, lack of understanding, fear, etc.), it is hard for them to teach math with confidence or enthusiasm to their students. However, when pre-service and in-service teacher education focuses on building understanding of math concepts as discovery learning, we can create new understanding and perspectives for the teachers. For instance, when we engage them in the same sort of real-life and constructivist experiences that they will use with their students, the teachers develop new understandings of mathematics and new teaching strategies simultaneously.

It is essential that the teachers engage in this learning firsthand, because it is a dramatic change from the way they learned mathematics. It is not easy to teach math in different ways;

however, when teachers experience the many new insights and perspectives inherent to this method, their math and teaching competencies are both advanced, even transformed.

Based on our understanding of constructivist math education with realistic learning, and our work with thousands of teachers at CUNY in Staten Island, NY, we have developed a specific, successful approach. We recognize that core goals of constructivist outcomes of modeling constructivist math education in teacher education include helping teachers:

- develop their mathematics understanding,
- become skilled facilitators of problem solving and learning, and
- ask questions of students continually, rather than dispensing information. (Lyublinskaya, I., & Kerekes, J. (2008) [13].

The next sections provide further information about the real-life applications and several specific examples.

- **Real-life Applications for Early Childhood Math Teaching and Learning**

3.2.1. Instructional Strategies and Findings

In the first phase of this sequence of activities, the teacher uses toys as math manipulatives, constructing stories and engaging students in internalizing number sense and problem solving. The following example reveals how a teacher may facilitate children's hands-on discoveries of number sense and problem solving in this manner.

3.2.1.1. Example 1: Real-life Context Number Sense

Materials needed: 2 die, or large sized playing cards

Overview of lesson: By rolling the die and asking questions such as, "Who has more?" Students learn number sense in a real-life context.

Detailed Overview: Children, individually or in pairs play with the die. In the process,

- They learn about **magnitude,** and recognize which number is bigger than the other.
- They develop **hierarchical inclusion** skills and determine several ways to identify the smaller number within the bigger number.
- They advance their **analytical skills** through questioning.
  - Examples, "How do you know?", "How many more?", etc.
  - Student responses involve engaging in any and all of the following
  - Demonstrating/showing their number concepts,
  - Paraphrasing their insights/observations

Teacher's script:
Students roll dice and eventually get 2 and 3

What do you have?
Student: 3
What do you have?
Student: 2
Who has more?
Student: I do
How many more?
Student: 1 more
(They are learning magnitude)

Students roll dice and eventually get 2 and 5

Can you see the two in my five?
Student shows
Can anyone see another two in the five?
Student shows
(They learn hierarchical inclusion)

Students roll dice and continue with different combinations

Questioning continues as above
(They learn more aspects of hierarchical inclusion in order to internalize the concept)

3.3.1.2. Example 2: Real-life Context: Math Concept– Making tens - 7 dice game

Materials needed: 7 dice (big foam is better), rug, 2 box of colored unifix cubes (200)
Overview of lesson: This lesson demonstrates one way young children can learn about basic addition and subtraction. They learn math concept of number sense without memorization.

Teacher's script:
Roll the 7 die.
Do not touch them.
Try to make as many tens as you can out of this combination.
You can use each dice only once.

Explain: If you find 2 3 3 1 1, or 4 3 2 1
Take the same number of unifix cubes as the number of dots on each choose dice in different color and with 1 to 1 correspondence put them on the top of the dots.
So, if you have 2 red colored unifix cubes on the first die, 3 blue, 3 green, 1 white, 1 pink. (The second example) Put all the unifix cube together.
What do you have? You have a rod with 2 red, 3 blue, 3 green, 1 white, 1 pink. Woo! How long your or? 10 unit long. What about the second sample? It is the same in length. What is different? How I got it. Now I have 4 purple, 3pink, 2black 1blue

Follow up: Next time "Creating your "10 Book""
Purpose: Students understand all the different combinations of 10 through working with them.
Material: 10, 1 inch x 1 inch squares strip glued on a sheet of paper**.**

Teacher's script: Color in the squares exactly the same way as you see the unifix cubes.
It is very important that you match the color of the square with the unifix cube.
What other color do you have?
Student: Yellow
What do you want to color with the yellow?
Student: Suns
Draw 3 yellow suns next to the yellow squares
What other color do you have?
Student: Blue
What do you want to color with the blue?
Student: Pencils

Draw 2 blue pencils next to blue squares
What other color do you have?
Student: Red
What do you want to color with the red?
Student: Apples
Draw 5 red apples next to red squares
How many do you have all together?
Student: It is 10
Very good.
Put the sheets together in a booklet. This is your “10 Book”.

The students understand all the different combinations of 10 through working with them.

3.3. Current Trends in USA Early Childhood Mathematics Education

During my time teaching math education in the USA, we have experienced several different national initiatives to advance student achievement. In my research and experience, these programs shift emphases every five to ten years. This changing context could be quite confusing for teachers who are prepared solely based on the curriculum focus of a specific point in time.

Instead, I find that focusing on helping the educators develop their skills in mathematics, problem solving, and teaching facilitation, provides a universal foundation which will be effective regardless of the changes which take place. If the faculty have a strong mathematical knowledge it enables them to master any new curriculum focus.

Currently the USA national K-12 (mandatory education) curriculum initiative is named the Common Core State Standards. Every area of K-12 (mandatory) education, such as Mathematics has Common Core State Standards (Common Core State Standards Initiative, 2012) [14]. In the USA, this initiative was formalized in 2008 when Arizona Gov. Janet Napolitano, who had been the 2006-07 chair of the National Governors Association, announced the precursor building blocks were in development (Bidwell, 2014)[16].

The Common Core State Standards is a national initiative to bring the entire nation into alignment with common educational objectives by discipline and grade level. There are several reasons that this has become important in the 21st century. Beneficial reasons cited for the Common Core State Standard include increased mobility among our workforce (teachers, students and parents move for work and other reasons), and having a more uniform curriculum allows greater transferability of their records, achievements and skills.

In addition, in comparison to prior initiatives, such as No Child Left Behind (NCLB), the Common Core State Standards provides more space for creativity and collaboration in learning (Long, 2013)[17]. This difference is seen in the fact that the teachers have technology, manipulatives, and professional development schools available. The focus with the Common Core State Standards also has shifted more to developing teaching and learning solutions among teachers. And within the classroom, the physical environment has changed. Instead of the lines of tables and chairs facing the front of the room, there are multiple learning centers and manipulatives available for these young mathematicians to use as needed. The focus has shifted from test preparation solely to different types of student group work, and developing multiple solutions through collaborative learning. We have seen a similar positive shift in the classroom environment, school expectations and teacher readiness for math education innovation during this time.

4. Summary

This auto-ethnographic study has provided critical reflection regarding my work with early childhood pre-service and in-service teachers in New York City. As I have been invested for over 20 years in supporting teachers to discover a love for mathematics and math education, I realize that while many changes have happened during that time, my roots in constructivism have been essential.

This paper describes how and why teachers can apply constructivist, realistic math education strategies with teachers of young children. Moreover, it illustrates how to facilitate the development of young mathematicians via the internalization of essential math concepts through creative play and in-depth questioning techniques. We have seen an essential shift in our K-12 classrooms, but more is needed. It begins with helping the teachers experience math in a new, genuine manner. In turn, they internalize math concepts, develop confidence and excitement, and can design classroom experiences which share these essential attributes with their students.

References

[1] Ellis, C. (2009). *Revision: Autoethnographic reflections on life and work*. Walnut Creek, CA: Left Coast Press.
[2] Hinchey, P. H. (2008). *Action research primer.* Thousand Oaks, CA: Sage Publications.
[3] Tournaki, N., Bae, Y. S., & Kerekes, J. (2008). Rekenrek. *Learning Disabilities: A Contemporary Journal 6* (2), 41-59.
[4] Streefland, L. (Ed.) (1991). *Realistic Mathematics Education in Primary School.* Utrecht, Holland: CD-beta Press, Freudenthal Institute.
[5] van Someren, M. W., Barnard, Y. F., & Sandberg, J. A. (1994). *The think aloud method: A practical guide to modelling cognitive processes.* London: Academic Press.
[6] Chang, H. (2008). *Autoethnography as method*. Walnut Creek, CA: Left Coast Press.
[7] Reed- Danahay. D. (1997). *Auto/Ethnography.* New York, NY: Berg.
[8] Taylor, M., Klein, E. J., & Abrams, L. (2014). Tensions of reimagining our roles as teacher educators in a third space: Revisiting a co/autoethnography through a faculty lens. *Studying Teacher Education: Journal of Self-Study of Teacher Education Practices, 10*(1), 3. doi:10.1080/17425964.2013.866549
[9] Izzo, A. (2006). *Research and reflection*. Charlotte, NC: Information Age.
[10] Freudenthal, H. (1987). *Theorievorming bij het wiskundeonderwijs-geraamte engereedschap*. Panamapost 5, 4-15.
[11] Creswell, J. (2014). *Research design* (4th ed.). Thousand Oaks, CA: Sage.
[12] Merriam, S. B. (2009). *Qualitative research* (3rd ed.). San Francisco, CA: Jossey-Bass.
[13] Lyublinskaya, I., & Kerekes, J. (2008). *Integrating Mathematics, Science and Technology in the Elementary Classroom.* Oceanside, NY: Whittier Publications, Inc.
[14] Common Core State Standards Initiative. (2012). *Common core state standards for mathematics*.
[15] Fosnot, C. T. & Dolk, M. (2001). *Young mathematicians at work.* Portsmouth, NH: Heinemann.
[16] Bidwell, A. (2014, Feb 27). The History of Common Core State Standards. US News and World Report. Retrieved from, http://www.usnews.com/news/special-reports/articles/2014/02/27/the-history-of-common-core-state-standards[
[17] Long, C. (2013, May 10). Six ways the common core is good for students. Retrieved from http://neatoday.org/2013/05/10/six-ways-the-common-core-is-good-for-students-2/