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PHOTOMATH: A BLESSING OR A CURSE

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Abstract
The use of mathematical software packages and applications becomes more and more natural for instructors and students as well. This process changes our tasks as instructors, the way of assessment we use, and even the mathematical behavior of students. The aim of this study is to show our experience about the spontaneous use of PhotoMath in class.

1. Introduction

We planned to consolidate the concept of logarithm during an elective course. We have the activity as a card game where students can use their calculator and a formula sheet with the properties of logarithm. However, during the game we realized that students pulled out their phones and started to take photos of the cards with logarithmic expressions. Why did they do this? The cards were not that nice…As a response to our question they showed us the application called PhotoMath on their phone.

2. PhotoMath

PhotoMath is a free mobile application capable of solving mathematical problems. Its first version appeared in 2014. The latest, version 7.5 was released in January 2021.

Figure 1. The Icon of PhotoMath

Compared to standard mathematical software packages Photomath is freely available on more than 30 languages and works without Wi-Fi. The novelty comes from its ease of use. It is enough to scan the problem using the phone's camera and the application gives the solution. While earlier versions could handle only printed formulae, recent editions recognize handwriting as well. The application identifies the problem, gives the solution and it even shows the step-by-step solution with explanations. Explanations can be requested at different levels: steps of the solutions, detailed description of each step depending on our needs (e.g. by solving an equation a step can be explained as "Divide both sides by this" but it also can be refined further thus we can get instructions on how the division can actually be performed). Multiple solutions are available. If graphical solution exists, the application will show that as well. It memorizes problems solved recently. A scientific calculator is also part of it.

PhotoMath handles basic arithmetic involving integers, fractions, decimals, powers, and roots. It solves linear equations and inequalities, deals with systems of equations, handles quadratic and trigonometric expressions, exponents, logarithms. It also capable of doing some higher mathematics such as complex numbers, matrices, differential and integral calculus.

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3. Literature review

The appearance of Photomath and similar products raises various questions on teachers' and students' side alike. Klinger [6] claimed that similar applications will contribute to the change of mathematics education. Despite their revolutionary nature only a few publications addressing the topic are available. We found that the papers available to us were addressing mainly two questions. They either analyze the classroom use of PhotoMath or they report about teachers' opinion and reactions.

PhotoMath was tried during in class teaching in the case of different topics for middle and high school students. In [5] the authors report about the use of the mobile application in the case of teaching elementary algebra. According to the study students achieved significantly better results when PhotoMath was used during the learning process. However, it is unclear whether the mobile application was used only for teaching or also for the assessment as well. A similar Malaysian study reported in [10] has also found improved performance in the case of algebraic equations. Mainly South American (Brazil, Peruvian, etc.) publications informed about the use of Photomath in high school level mathematics. Here the application was used as a calculator. Thus, it served mainly for checking solutions, evaluating complex expressions in the case of previously known problems. The tasks investigated by students were traditional problems not specifically designed for Photomath. The application probably served here as a graphical calculator. Most students have already a mobile phone, the application is free, and the authors emphasized that it does not require a wi-fi connection. Hence, even those students can benefit its use who otherwise would not have a chance to gain access to CAS or educational software packages such as GeoGebra.

Based on the publication available ([4], [6], [7], [9]) we can generally conclude that the opinion of both teachers and students are positive. Klinger has found in [7] that German pupils used the application mainly for solving homework and preparing for tests. Their attitude toward the use of it was overly positive. A Jordanian study surveyed the opinion of teachers of the application. In [4] the authors have found that the acceptance of the use of PhotoMath correlates with the level of education of teachers. Higher level of education indicated a significantly higher support. Korenová és Veress-Bágyi declares a generally positive attitude towards Photomath in [9], the authors felt that there is room for improvement as similar applications could offer a wider scope of use to help university students than what is the current practice in Central Europe.

Besides these overly positive opinions we could hardly find papers with a more critical tone. One of those is [2], an American study where the author addresses the issue of potential cheating in homework and exams as well. A good summary of other critical opinions of teachers can be found in [11]. The authors, Webel and Otten posed the question whether we should consider PhotoMath as an ideal instrument for assisting learning or as a perfect tool for cheating. They offered three different reactions towards the application, such as completely ban its use, restrict access to it, and consider a different approach to teaching mathematics. They discussed the difficulties and opportunities of all three possibilities. They emphasized that it is important to redefine our didactic goals according to the technology available. They mentioned equation (1) as an example for a different division of labor while solving mathematical problems. The version of PhotoMath available by the authors when [11] was written offered a long algorithmic solution for (1) containing 12 steps. In this solution the application suggested to open the brackets first and solve the equation containing fractional coefficients. Opposed to this solution the intuitive and obvious approach to the problem is to divide both sides by $3/8$ and solve the remaining $2x-5=1$ easily. (The solution suggested in the latest version of Photomath offers now the optimal simple solution).

$$\frac{3}{8}(2x-5)=\frac{3}{8} \tag{1}$$

The authors saw a great potential in the discussion of similar problems on whether and how we should use technology. They argued that instead of or besides completely or partially prohibit the use of similar applications they can offer a new scope in mathematics education. Webel and Otten suggested that instead of traditional problems we should design specific tasks using the opportunity offered by technology.

Klinger and Schüler-Meyer recommend such tasks in [8]. They suggested the investigation of the relationship between zeros of the quadratic function and the number of real roots of the corresponding quadratic equation. They also described a problem intended to deepen differentiation rules. The authors also discussed the advantages and disadvantages of the use of mobile application versus traditional computer algebra systems. They mentioned that graphing more functions in the same coordinate system as a very useful tool that cannot be carried out by a mobile application. They also stressed that it is very convenient to only scan the problems but in certain cases PhotoMath requires using the appropriate notation (e.g. d/dx for differentiation).

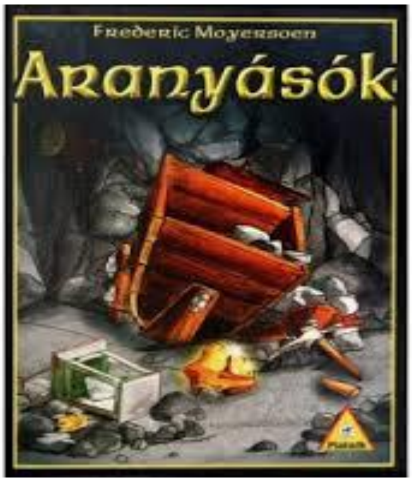


Figure 2. The card game Saboteurs

4. The Saboteurs card game and its mathematical version

A course titled Games and Mathematics (Játék és Matematika) is offered since 2015 at the University of John von Neumann. The course is based on the mathematical component of a complex remedial program. The main goal of the course and the remedial program alike was to develop students' mathematical competencies using cooperative techniques through games. In most cases we used commercial card and board games and filled them with mathematical content.

The most appropriate game for this purpose was the card game Saboteur. The rules of the game are easy to understand and to adapt to various purposes. The original version is short, does not require special equipment and even 10 players can participate. It is a fun game with considerable social interactions, and it relies heavily on cooperation. The players are divided randomly into two groups, they became either gold miners or saboteurs. Their role is determined by a card drawn at the beginning of the game and it remains secret during the game. The other players can guess and infer one's role based on his or her actions during the game. The aim of the gold miners is to find the gold hidden under one of three cards by building an uninterrupted path to the treasure. In case they find it, the group wins. Saboteus try to prevent this by adding dead ends and mis-ways to the path. If the players run out of cards and the gold miners could still not find the gold nugget, the saboteurs win. Players either build a path using path cards or use action cards to help or hinder their own or other players' job. They can use action cards to figure out the exact


Figure 3. An illustration of the game Sabotuers

Figure 4. The card game Math-teurs

place of the gold nuggets, they can hinder someone by breaking his or her tool or can fix the tool and give the chance to the person to build again. It is also possible to remove a path-card from the existing maze by using the card of a rockfall. Apart from building or playing an action card one can also pass a card if a player cannot or does not want to play it. After one's action a player draws a card from the deck as long it still contains cards. The game ends when the gold miners reach the gold nugget, or the draw deck runs out of cards.

We have customized this original game by changing the path cards to "number cards". Instead of non-interrupted path players should build a path by forming an increasing sequence of numbers throughout the neighboring cards from the start to the gold nugget. Players denote the relations between two cards by placing a match between the two numbers. The match head points to the greater number. We called the new version of Saboteurs as Math-teurs.

We created number cards to various mathematical topics, such as integers, fractions, trigonometric, exponential, and logarithmic expression, complex numbers. Among those we focus only on logarithmic expressions as this was used during the game we have analyzed. By creating the 40 number cards we kept in mind that the mathematical task was to order logarithmic expressions. Although we encouraged students to evaluate the expressions at the beginning, the ultimate goal of the activity was to reach the level when students can compare two expressions based on the properties of the logarithm function. Thus, instead of calculating the exact values of the expressions, students should know how to rewrite the expressions using only one logarithm. Furthermore, they should also realize that the monotonicity of the logarithm function assures that it is enough to compare only the arguments of the logarithmic expressions. In order to facilitate this process, we have included a few expressions where the exact value cannot be calculated using the definition and the logarithm rules.

5. Analysis

We recorded a card game of Math-teurs played by six students in one of the classes of the course Games and Mathematics. Students already knew the game as in a previous lesson they had played the exponential version of Math-teurs. They had access to the cards, a matchbox, and a formula sheet before the actual game. As the concept of logarithm was not knew to students, we just shortly reviewed the definition and the logarithm rules available also in the formula sheet before the game.

In previous years we observed that the groups went through always the same four phases throughout the game reaching the third or the fourth of the following levels:

1. Calculating the value of an expression using a calculator
2. Calculating the value of an expression using the definition of logarithm
3. Calculating the value of an expression using logarithm rules
4. Comparing expressions using the monotonicity of the logarithm function

In the game recorded students used PhotoMath only once. We analyze this part of the game in this section. In the analysis we would like to compare whether there is a difference between the mathematical behavior of students before and during the use of the mobile application. For the analysis we mainly used the transcript of the recording.

Expression (2) elicited the use of PhotoMath. This was the most complicated expression among the 40 we have prepared. The expression cannot be evaluated by using the definition or logarithm rules, and it also required the use of various logarithm properties.

$$3 * log_2 \frac{3}{2^3} - log_2 2 \tag{2}$$

Students already met an expression $log_2 3$ at the beginning of the game when they already started to get acquainted with the cards and estimated it correctly. They tried to use the same method here as well, namely estimating the expression of each term, but this this time it was incorrect. In the following section we show the step-by-step description on how students reached the answer. For the detailed analysis we use excerpts from the transcript. (We translated these excerpts to English).

Student 1: …*3-1, right? Or you should get something similar in case you calculate it.*

They also try applying logarithm rules by writing the whole expression using only one logarithm and estimate the answer this way. Unfortunately, this trial was wrong as they incorrectly applied the quotient rule.

> Student 5: *You subtract 2 from the 3/8 to the power of 3. Thus, it is rather negative.*

They could not deal with it and did not find the mistake in the argument.

In the meantime, one of them tries to convince the others to consider applying the logarithm rules:

> Student 6: *…is it 3 to the power of 3 divided by 2 to the power of 10?*
> Later: …*27 divided by 1024*.
> Finally: *You bring this three here, then it is 3 to the power of 3 and this will be 2 to the power of 9, and you bring this here then. And then it will be 2 to the power of 10…*

It still did not help the evaluation

> According to Student 1: *We should rather wait 'till (Student's name) tells us.*
> Thus, the need for technology is elicited by the unsuccessful evaluation of the expression in question.
> They first use a calculator:
> Student 2: *Well (student's name) typed in, but he got -3, thus it seems that he mistyped it.*
> They did not accept -3 as an answer. PhotoMath appears here as students' last hope.
> Student 3 takes out his phone and asserts: *I will tell you the answer.*

They first evaluate an expression they already know the result of: $log_2 1$. This serves as a verification whether PhotoMath calculates correctly.

> After using the application Student 3 asked: *Which one do we know for sure? log 1? It's a miracle.*
> Student 5 reacted: *It shows 0 immediately.*
> Since the answer is correct, they scanned the "complicated" expression as well.
> Student 2: *Now put in that long one.*
> Student 4 questioned the answer given by the application: *It is surely not.*
> Student 3 assumed wrong scan: *I guess it has omitted a part.*
> Therefore, he rescanned it: *It shows the same again*.
> Seeing the same result Student 2 checked the steps of the solution on PhotoMath: *It is split correctly.*
> Student 6 found the answer correct: *It can be right*.
> Student 1 accepted -5,2 as an accurate answer: *Does not matter. We will rely on it.*
> Student 3: *We cannot make any progress otherwise.*

In this 4-minute playtime we could identify four different types of mathematical activity described below. After defining the four categories we present the color-coded data in a graph as a function of time.

1) Demand
   Demand for help. In this case help refers to the use of technology (calculator/ mobile application).
   Shortly after card (2) appears Student 2 suggests: *Then Wolfram Alpha*[*]*?*
   After finally evaluating the card using PhotoMath Student 1 showed the demand for its further use: *Do not dare to close that App.*
   Coded green in Figure 5.
2) Question

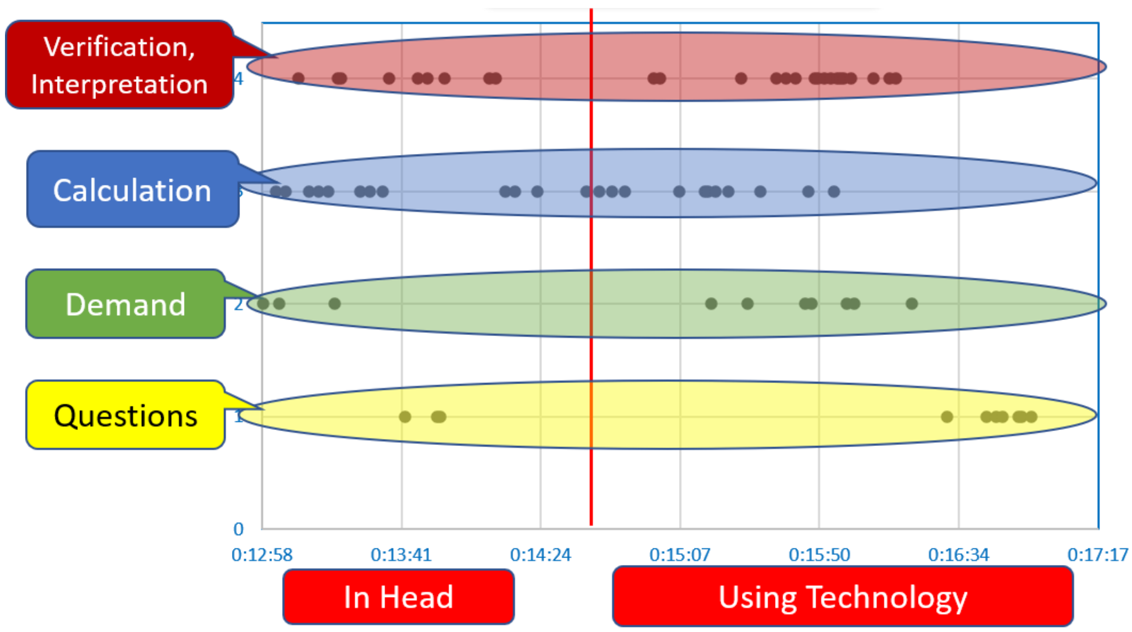
[*] Wolfram Alpha is a computational knowledge engine providing online answers for factual and computational questions. It was based on the company's former product, Wolfram Mathematica, a computer algebra system used mainly for symbolic and numerical calculations and visualization.

Posing questions to the group, or demanding reassurance from the others.
Student 5: *Wait! What did he say? 5,2?*
Coded yellow in Figure 5.

3) Calculation
Calculation in head, using calculator or PhotoMath. The category refers to the actual process of calculating.
Coded blue in Figure 5.
4) Verification and interpretation
The category refers to all the activities involving verification and interpretation; thus, it contains questioning the results obtained, estimating logarithmic expressions with or without the help of technology, checking the answers, interpreting solutions.
Coded red in Figure 5.

The diagram in Figure 5. shows the appropriate mathematical behavior (numbered by 1-4 and color-coded accordingly) as a function of time. The red vertical line shows the time when students started to use PhotoMath.

Figure 5. The analysis of the game



Here we summarize the differences between the calculations in head and those performed with the help of technology.

1. Verifying solutions takes on the role of estimation. This verification happens often, students also check the steps of the solution offered by PhotoMath and constantly interpret the suggested answer.
2. Interpreting the solutions plays a more significant role than before. The answers obtained as a result of a step-by-step process before. Technology changed this process to a tool for verification, whereas student performed the calculation after the results were given using technology and they confirmed the answer this way.
3. Questioning the results and the demand for verification appears. It is interesting that students do not trust PhotoMath. While incorrect or surprising result is attributed to syntactic errors in the case of calculators, it is explained as the error of the application in the case of PhotoMath.
4. There is constant need for using technology. Students constantly verify the steps of the new ideas arising and recalculate the results accordingly.

6. Summary

The appearance of PhotoMath and similar applications will certainly influence the tasks of future mathematics classes and assessments. We have found that students mathematical behavior changed while using PhotoMath as they tended to question the results obtained with the help of the application, they constantly checked their solutions, and interpreted the obtained answers. It would be interesting to investigate the consequences of this behavior on teaching and learning mathematics.

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