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OPTIMAL TILT ANGLE OF PLANE SUNLIGHT-COLLECTORS IN HUNGARY

MAGYARORSZÁGON HASZNÁLT SÍK NAPKOLLEKTOROK OPTIMÁLIS DŐLÉSSZÖGE

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Abstract
Solar radiation is a function of various parameters viz., solar altitude, location (latitude and longitude), site altitude, albedo, atmospheric transmittance and cloudiness. However, clear sky solar radiation is independent of cloudiness index. Solar radiation (global, diffuse and direct) variation on tilted surfaces is presently calculated using data of ASHRAE clear-sky model*.*

Összefoglalás
A napsugárzás mértéke több tényező függvénye: úm. a Nap horizont feletti magassága, földrajzi hely, az albdedo, az atmoszféra áteresztőképessége és a felhőzet, bár teljesen tiszta égboltesetén a felhőzetnek nincs befolyásoló hatása. A különböző ferdeségi szögben elhelyezett sík napkollektorra eső napsugárzás változását vizsgálja a dolgozat az ASHRAE clear-sky model *adatainak felhasználásával.*

1 Introduction

Solar energy is a treasure resource in nature and plays an important role in power supply in future resulting from the shortage of fossil fuels. Knowledge of global solar radiation and its components (direct and diffuse solar radiation) is required for analysis of solar energy conversion systems. In many solar energy applications, the solar energy incident on the surface of the earth is an essential requirement for assessment of the performance and evaluation of efficiency of solar energy systems. Also, detailed analysis of solar radiation data is necessary to estimate solar energy potential of the site [1]. Solar energy can be utilized directly through a variety of devices such as solar collector. Generally, in the northern hemisphere the best azimuth is due south (facing equator), but the tilt angle varies with factors such as the geographic latitude, climate condition and utilization period of time.

Renewable energy obtained from the sun is very important because of the fact that it is free and environment-friendly. The importance of detailed knowledge of solar radiation received from the sun at a site in the design and selection of solar devices cannot be overstated. In order to optimize solar isolation on solar collectors, appropriate method to determine solar tilt angles at any given time is essential to increase the efficiencies of the collectors and that of the devices connected to them. The position of the earth relative to the sun changes with time; the change must be monitored adequately in order to increase the amount of energy being received by solar devices.

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2 Factors affecting the collector tilt angle

Solar radiation incident on a collector is composed of three components, i.e. the direct beam, diffusion and reflection from the ground, which has different dependence on the slope of collector, the sum of these three components is called global radiation. Installing a collector properly can enhance its application benefit because the amount of radiation flux incident upon the collector is mainly affected by the azimuth and tilt angles that it is installed.

2.1 Direct beam

In case of clear sky the direct radiation from the Sun to a perpendicular surface depends on the elevation angle β of the Sun. Using the ASHRAE clear-sky model [2] to estimate the global solar radiation on perpendicular surfaces is:

$$S_{KM} = \frac{a}{\exp\frac{b}{\sin\beta}} \tag{1}$$

where
- S_{KM} – radiation sum of direct radiation [W m^{-2}]
- a – ASHARE empirical constant (see table 1) [W m^{-2}]
- b – ASHARE empirical constant (see table 1) [-]
- β – the elevation angle of the Sun [grad]

Table 1. Constants for ASHRAE equations (1) and (5) for the 21st day of each month [2]

Months	a [w m^{-2}]	b [-]	c [-]
Jan	*1230*	*0.142*	*0.058*
Feb	*1214*	*0.144*	*0.060*
Mar	*1186*	*0.156*	*0.071*
Apr	*1135*	*0.180*	*0.097*
May	*1104*	*0.196*	*0.121*
Jun	*1088*	*0.205*	*0.134*
Jul	*1085*	*0.207*	*0.136*
Aug	*1107*	*0.201*	*0.122*
Sep	*1151*	*0.177*	*0.092*
Oct	*1192*	*0.160*	*0.073*
Nov	*1220*	*0.149*	*0.063*
Dec	*1233*	*0.142*	*0.057*

The radiated power per unit area from the S_{KM} direct radiation is:

$$\dot{q}_K = k \cdot S_{KM} \cdot \cos\eta(\beta) \tag{2}$$

where
- $\dot{q}_K$ – radiated power per unit area [W m^{-2}]
- k – the transmission factor [-]

Based on measurement data of *Godbey et al* [3], the maximal value of k is at $\eta=0^0$ $k_{max} = 0.83$ for 0.1 mm thick polyethylene film; and in angle range between $0^0 \geq \eta \geq 70^0$ formed according to the following formula:

$$k = k_{max} - 0.000912 \cdot \exp\ 0.0936 \cdot \eta \tag{3}$$

The total direct radiation for a whole day q_K [J]

$$q_K = 2\int_0^{\beta_0} k \cdot S_{KM} \cdot \cos\ \eta(\beta)d\beta \tag{4}$$

2.1 Diffuse radiation

The diffuse radiation for a horizontal surface is

$$S_{SZV} = c \cdot S_{KM} \tag{5}$$

where *c* – ASHARE empirical constant (see Table 1.) [-]

The radiated power per unit area from the S_{SZV} diffuse radiation is:

$$\dot{q}_{SZV} = \frac{k \cdot S_{szv} \cdot (1+\cos\varphi)}{2} \tag{6}$$

and so the total diffuse radiation energy for a whole day q_{SZ} [J]

$$q_{SZ} = \int_0^{\beta_0} k \cdot c \cdot S_{SZV} \cdot (1 + \cos\ \varphi)\, d\beta \tag{7}$$

2.2 Reflected radiation

On the horizontal surface located in front of the collector fall on the radiation S_{TV}

$$S_{TV} = S_{SZV} + S_{KM} \cdot \sin\beta \tag{8}$$

The radiated power per unit area of the collector from the reflected radiation is

$$\dot{q}_V = \frac{r \cdot k \cdot S_{TV} \cdot (1-\cos\varphi)}{2} \tag{9}$$

where *r* – the reflectance factor of the horizontal surface located in front of the collector (by grass *r* = 0.2; [4]) [-]

and so the total reflected radiation energy for a whole day q_V [J]

$$q_V = \int_0^{\beta_0} r \cdot k \cdot S_{TV} \cdot (1 - \cos\ \varphi)\, d\beta \tag{10}$$

The total energy falling on collector surface-unit q_T [J] is

$$q_T = q_K + q_{SZ} + q_V \tag{11}$$

2.3 The impact position of the Sun

The rates of angles β and η as well as factor *k* are depending of the current position of the Sun. To track the Sun's rotation should be used τ the sunshine hour angle. The value this angle is in case that the midday Sun *0* degree, calculated from 15 degrees per hour. The height of the current Sun is determined by the location under investigation latitude λ, the angular position of the Sun from the equator δ, and the hour angle τ.

The β angle of incidence in midday from the ABC triange (Fig 1):

$$90^0 - \beta = \lambda - \delta \tag{12}$$
$$\cos(90^0 - \beta) = \cos\lambda \cdot \cos\delta + \sin\lambda \cdot \sin\delta$$

where λ – is the latitude of the site (Kecskemét 46.8963711^0) [degree]

$$\sin\beta = \cos\lambda \cdot \cos\delta + \sin\lambda \cdot \sin\delta \tag{13}$$

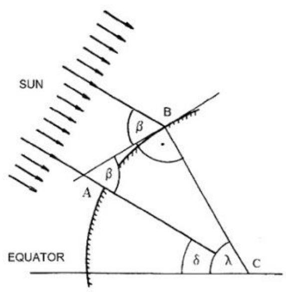

Figure 1. Features of the β angle of incidence depending on λ geographic latitude

Taking into account the hour angle τ:

$$\sin\beta = \cos\lambda \cdot \cos\delta \cdot \cos\tau + \sin\lambda \cdot \sin\delta \text{ and so } \beta = \sin^{-1}(\cos\lambda \cdot \cos\delta \cdot \cos\tau + \sin\lambda \cdot \sin\delta) \quad (14)$$

The solar azimuth angle γ is

$$\gamma = \sin^{-1}\frac{\cos\delta \cdot \sin\tau}{\cos\beta} \quad (15)$$

The angle η of incidence using the formers as (Fig 2.):

$$\sin(90^0 - \eta) = \sin(\varphi + \gamma); \quad \eta = \cos^{-1}(\sin\varphi \cdot \cos\gamma + \cos\varphi \cdot \sin\gamma) \quad (16)$$

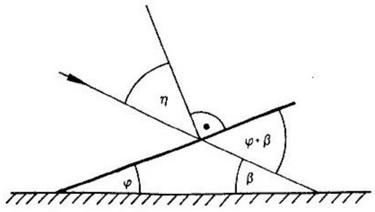

Figure 2. Features of the η angle of incidence depending on the φ angular offset of the collector

2.4 Determining the optimum collector tilt angle

The optimum value of collector tilt angle is the angle wherein the amount of direct, diffuse and reflected radiation – absorbed by the collector – is the maximal. The maximum value of the total radiation can be determined in a way that – for the same calendar date – it will be calculated the values Q_T for different tilt angle step by step from 0 degree to 90 degree. The optimum tilt angle φ is that one that fit to the maximum value of Q_T.

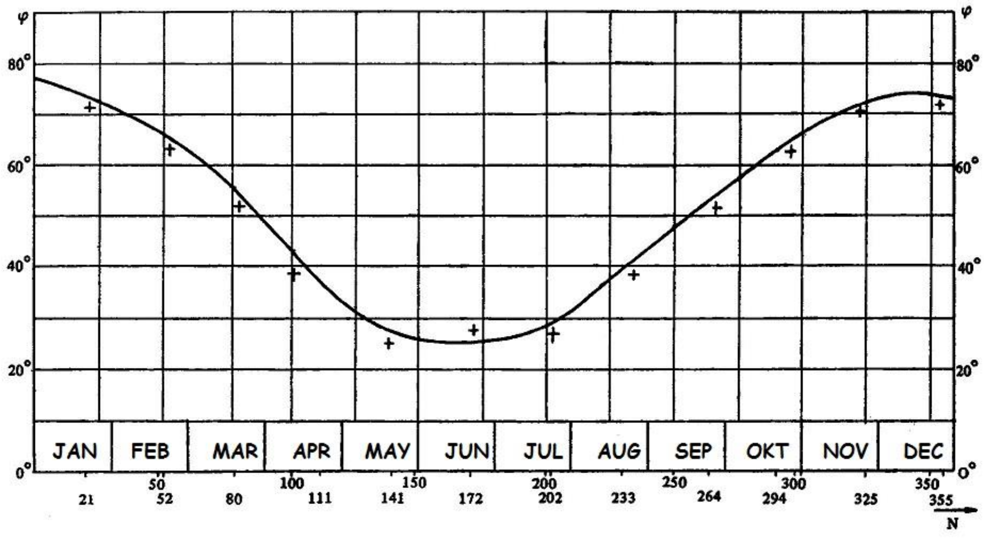

Figure 3. Features o f the optimum angular offset in the course o f the year

3 Results and evaluation

It were calculated the optimal tilt angle for the 21st day of each month. The results are shown in the time, [*the day of the year starting from the first of January N*] and *tilt angle* [φ, degree] coordinate system, the equation of the looked for in the approximate function that was fitted to the coordinate points (Figure 4.). The south-facing flat solar collectors optimal tilt angle varies from day to day. By fitting an optimal tilt angle values specified day of the 21 months depending n any day of the year for optimal tilt angle can be easily determined:

$$\varphi = 49 + 25 \cdot \cos 0.9863(\mathrm{N} + 10) \quad (17)$$

Table 2. Recommended time intervals and angle figures for mounting the sunlight-collectors

PERIOD	φ [degree]	PERIOD	φ [degree]
12th Feb – 13th Mar	60	9th Jul – 14th Aug	30
13th Mar – 2nd Apr	50	14th Aug – 2nd Sep	40
2nd Apr – 24th Apr	40	2nd Sep – 29th Sep	50
24th Apr – 27th May	30	29th Sep – 28th Oct	60
27th May – 9th Jul	25	28th Oct – 2nd Feb	70

Since the collectors daily claims uneconomical and because -variation ± 5° with an optimum angle is no greater than the Q_T value of 1% was judged to be an error, it is sufficient to produce after collectors 10° intervals. Using the graph of Figure 4 can be determined as a function of time intervals for each angle values (Table 2).

4 Conclusion

In this paper, it was developed a mathematical model for determining the optimal tilt angle for maximizing the total solar radiation incident on an inclined surface, using the global solar radiation incident on a horizontal surface.

Based on the analysis presented, it is recommended that the ASHRAE clear-sky model can be used for estimation of clear sky solar radiation in Hungary. Solar radiation incident can be easily analyzed using the ASHRAE model. Also, the components of global solar radiation (beam, diffuse and reflected) can be calculated separately. Further, clear sky radiation can also form the basis for evaluation of various sky conditions. The paper also presents a regression model that allows the prediction of the periodical optimal tilt angle for maximum solar radiation collection for sites in the Hungarian geographical latitude.

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