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TEMPERATURE INCREASE IN A LASER-BEAM HEAT-TREATMENT OF METALS

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Abstract
Laser-beam heat-treatment of a plane metal surface is modeled in a constant material parameters approximation. A relatively simple formula is created for the maximal temperature increase as a function of material parameters, the power and the width of the light-beam, and velocity of the sweep. This constant parameter approximation cannot be very precise but it is hoped to be useful in practice of laser surface heating experiments just to get an approximate value of the surface temperature increase.

1 Introduction

Frequently, bulky and surface properties of metal should be different. Laser heat treatment (also known as laser surface hardening) can obtain approximately 10 Rockwell C higher hardness than any other hardening method available on the market today [1]. This an be achieved by focusing a well defined beam of intense laser light on the component to be laser hardened, combined with rapid self-quenching. This method is important (see for example [1, 2]), and its fundamentals are well known [3, 4].

In this paper I give a simple formula for temperature increase on a metal surface when it is irradiated with a scanning laser beam. This formula is based on an a very crude approximation of real processes: we assume that all the parameters (heat conductivity, density, specific heat) of the metal are constant during the heating-cooling process. I present here also the derivation of the basic equations because it is quite instructive, but I know that very similar calculations were published as early as in 1977 [3]. My real work was just to create a formula for a fast evaluation of the temperature increase in the vicinity of the metal surface. This formula cannot be very precise because its' derivation disregard the effect of reradiation of heat and the variation of material parameters of the heated metal. Its precision cannot be enugh for precise temperature calibrations for laser heat treatment [5]. However, I hope that the presented formula will be useful for experimentalist heating of metal surfaces with scanning laser beams just to guess the current temperature increase.

2 The model of surface heating of a metals

The air-metal interface is on the x-y plane, the metal fills the lower halfspace (z<0). The laser beam of width w_0 and of power P arrive orthogonally onto the air-metal interface, it moves along the surface with a constant speed of v. The heat is generated by absorption of the power of the laser beam. Let us suppose a laser beam with Gaussian lateral intensity distribution with a light-beam-width parameter w_0 (see fig. 1) [6]. The heat-source, f is concentrated on the $z = 0$ plane, because of the high absorptivity of metals:

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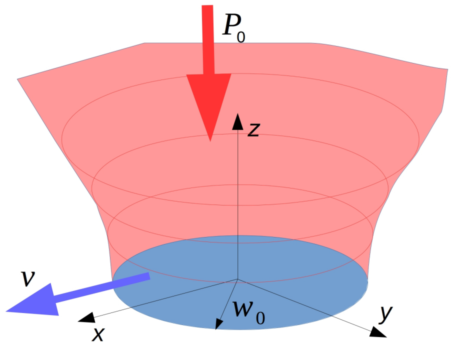


Figure 1. The model of surface heating by a scanning laser beam

$$f\left(x,y,z,t\right)=\frac{P}{\pi\,w_0^2}\,\exp\left(-\frac{\left(x-v\,t\right)^2+y^2}{w_0^2}\right)\,\delta\left(z\right) \tag{1}$$

where P_0 is the power of the laser beam, P is the absorbed power, $P=\tau\,P_0$, and τ is the transmissivity of the air-metal interface (it can be regarded also as absorption coefficient because the light trasnmitted by the interface is absorbed in the bulk material); $\delta\left(z\right)$ denote the Dirac-delta "function" which can be used here because metals have high absorption coefficient values, that is the incident light is absorbed on the very close ($\mu\mathrm{m}$ or less) vicinity of the surface.

The heat expansion is governd by the following equation:

$$c\,\varrho\,\frac{\partial\,\Delta T}{\partial\,t}-\lambda\,\frac{\partial^2\,\Delta T}{\partial\,x^2}+\frac{\partial^2\,\Delta T}{\partial\,y^2}+\frac{\partial^2\,\Delta T}{\partial\,z^2}=f \quad , \tag{2}$$

where $\Delta T=\Delta T\left(x,y,z,t\right)$ is the sought temperature-increase distribution in the metal in the lower half-space below the $z=0$ plane; c is the specific heat, ρ is the density and λ is the heat conducting coefficient of the metal.

In this paper constant values are suppose for the material constants (τ, c, ρ, λ).

This inhomogenious heat-diffusion equation will be solved using it's Green-function.

In the next chapter an expressive derivation is given of the Green-function of the homogeniuos space. The temperature distribution induced by the absorption of the scanning laser beam (see eq. (1)) is determined in section 4.

3 Self-similar diffusion of a Gaussian heat distribution

Let us suppose that the space filled width a material homogenously. The homogeneous heat diffusion equation

$$c\,\varrho\,\frac{\partial\,\Delta T}{\partial\,t}-\lambda\,\frac{\partial^2\,\Delta T}{\partial\,x^2}+\frac{\partial^2\,\Delta T}{\partial\,y^2}+\frac{\partial^2\,\Delta T}{\partial\,z^2}=0 \tag{3}$$

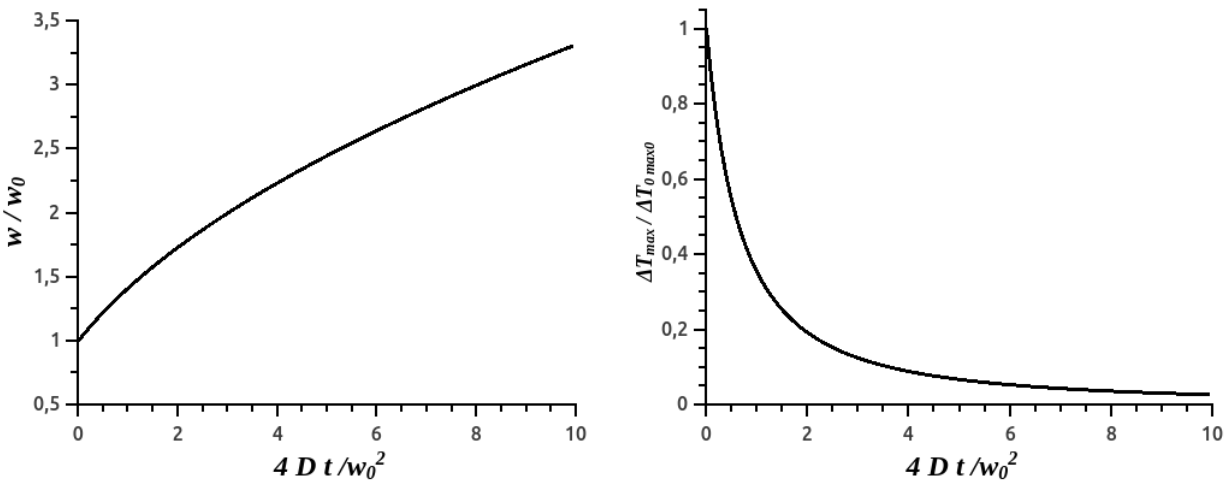
can be shown to have a self-similar solution ($w=w\left(t\right)$):

$$\Delta T\left(x,y,z,t\right)=\frac{E}{c\,\varrho}\,\frac{1}{\pi^{\frac{3}{2}}\,w^3}\,\exp\left(-\frac{x^2+y^2+z^2}{w^2}\right) \quad , \tag{4}$$

where the temperature distribution is normalized the way that the energy of the temperature increase is E:

$$c\,\varrho\int\Delta T\,\mathrm{d}V=E \quad . \tag{5}$$

The width of the distribution increases:



a) b)

Figure 2. (a): The speed of growing of the width of the distribution decreases in time. (b): The speed of the decrease of temperature departure also decreases as time rolls on (dimensionless time variable is used on the figures)

$$w\left(t\right)=\sqrt{w_0^2+4\,D\,t} \quad , \tag{6}$$

where $D=\frac{\lambda}{c\,\rho}$ diffusion constant of the material.

The width of the temperature departure (Gaussian-shaped) distribution increases with a speed decreased as the ditribution become bigger (see Fig. 2.a). The maximal temperature departure (at the origin of the coordinate system) can be seen on Fig. 2.b decreases slower as the time elapses, that is the size of the distribution gets bigger. This also means that a smaller hot spot cools more quickly.

The eq. (4) gives a solution that cloud be started from infinetesimal width of the distribution, that can be regarded as a Green-function of the heat conduction equation (eq. (3)).

4 Determination of the temperature distribution with the help of the Green-function

The Green-function G_0 of the whole space can be read from the above solution:

$$G_0\left(x,y,z,t\right)=\frac{1}{c\,\varrho\,\left(4\,\pi\,D\,t\right)^{\frac{3}{2}}}\,\exp\left(-\frac{x^2+y^2+z^2}{4\,D\,t}\right)\,\theta\left(t\right) \quad . \tag{7}$$

where $\theta\left(t\right)$ is for the delta function of Heaviside.

The same heat packet on the boundary of the half space (in the model of Sec. 2) results double temperature-increase in the lower half-space $G=2\,G_0$. The temperature distribution can be obtained with the help of the Green-function of the heat diffusion equation in the lower half space, which is the temperature-increase distribution obtained for a unit energy excitation in the origin at $t=0$:

$$\Delta T\left(x,y,z,t\right)=\int_{-\infty}^{\infty}\mathrm{d}t'\,\iiint\mathrm{d}V'\;f\left(x',y',z',t'\right)\;G\left(x-x',y-y',z-z',t-t'\right) \quad , \tag{8}$$

where using G is in close connection to the fact that the sources are on the boundary of the lower half-space.

The integrals (except one) can be evaluated by the help of the so called Siegman-integral [6]:

$$\int_{-\infty}^{\infty} \exp\left(-a\,t^2 - 2\,b\,t\right)\,\mathrm{d}t = \sqrt{\frac{\pi}{a}}\,\exp\left(\frac{b^2}{a}\right) \tag{9}$$

The solution gets simpler with the new variable $u = x - v\,t$ instead of x, the solution can be given in the coordinate system moving with the exciting laser beam:

$$\Delta T\left(u, y, z, t = 0\right) = \frac{2\,P}{c\,\varrho\,\pi^{\frac{3}{2}}} \int_0^{\infty} \frac{\mathrm{d}t'}{\left(w_0^2 + 4\,D\,t'\right)\,\sqrt{4\,D\,t'}} \cdot \exp\left(-\frac{\left(u + v\,t'\right)^2 + y^2}{\left(w_0^2 + 4\,D\,t'\right)} - \frac{z^2}{4\,D\,t'}\right) \quad . \tag{10}$$

If we introduce the following dimensionless variables

$$\xi = \frac{u}{w_0}\,, \quad \eta = \frac{y}{w_0}\,, \quad \zeta = \frac{z}{w_0}\,, \quad \vartheta = \frac{t'\,D}{w_0^2}\,, \quad \kappa = \frac{v\,w_0}{D} \tag{11}$$

the temperature departure, ΔT can be obtained in a simpler from:

$$\Delta T\left(\xi, \eta, \zeta\right) = \frac{P}{\pi^{\frac{3}{2}}\,\lambda\,w_0} \int_0^{\infty} \frac{\mathrm{d}\vartheta}{\left(1 + 4\,\vartheta\right)\,\sqrt{\vartheta}}\,\exp\left[-\frac{\left(\xi + \kappa\,\vartheta\right)^2 + \eta^2}{1 + 4\,\vartheta} - \frac{\zeta^2}{4\,\vartheta}\right] \quad . \tag{12}$$

The above is an improprius integral: the integrand is not defined in $\vartheta = 0$. It can be transformed to a definite integral with a new variable $s = 2\,\sqrt{\vartheta}$ ($4\,\vartheta = s^2$, $ds = \frac{d\vartheta}{\sqrt{\vartheta}}$):

$$\Delta T\left(\xi, \eta, \zeta\right) = \frac{P}{\pi^{\frac{3}{2}}\,\lambda\,w_0} \int_0^{\infty} \frac{\mathrm{d}s}{\left(1 + s^2\right)}\,\exp\left[-\frac{\left(\xi + \kappa\,s^2/4\right)^2 + \eta^2}{1 + s^2} - \frac{\zeta^2}{s^2}\right] \quad . \tag{13}$$

With this simple integral the temperature deviation value in any position can be calculated. It is worth to notice that this solution is valid if all the parameters of the medium is absolutly constant (see Sec. 2).

5 Simple formula for maximal temperature increase

The maximal temperature increase value can be well approximated by the value at the central point of the laser beam on the surface where $\xi = 0\,, \quad \eta = 0\,, \quad \zeta = 0$:

$$\Delta T_{max}\left(P, \lambda, w_0, \kappa\right) = \frac{P}{\lambda\,w_0}\,I\left(\kappa\right) \quad , \tag{14}$$

$$I\left(\kappa\right) = \frac{1}{\pi^{\frac{3}{2}}} \int_0^{\infty} \frac{\mathrm{d}s}{\left(1 + s^2\right)}\,\exp\left[-\frac{\kappa^2\,s^4}{16\,\left(1 + s^2\right)}\right] \quad . \tag{15}$$

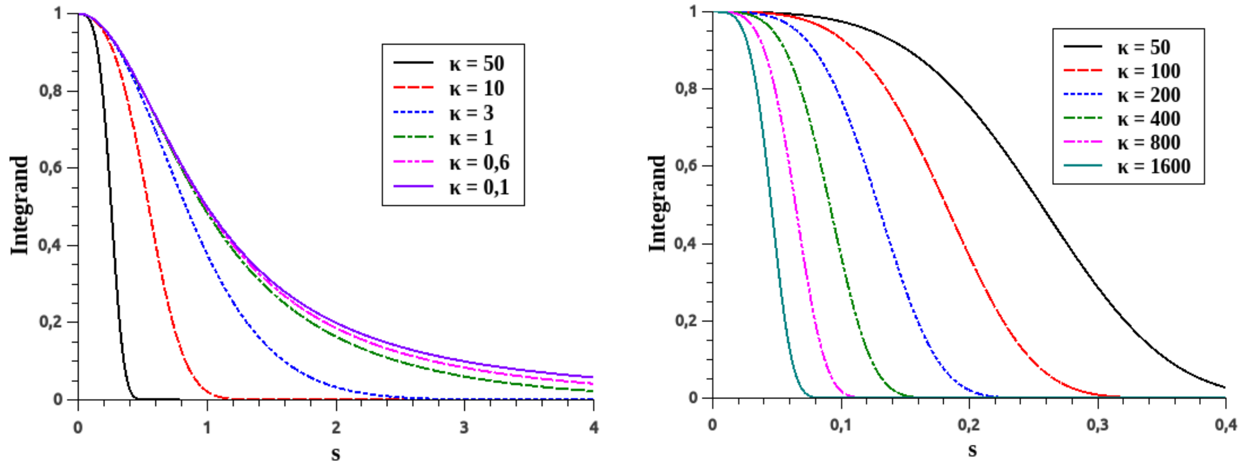
From the practical point of view it is important to find the useful parameter range in κ. Moderate values for steal are the following (0.1 m/s = 6 m/min, $c_{steal} \approx 470\,\mathrm{J/kg\,K}$, $\lambda \approx 50\,\mathrm{W/m\,K}$, $\rho \approx 7800\,\mathrm{kg/m^3}$):

$$D = 13.7 \cdot 10^{-6}\,\frac{\mathrm{m}^2}{\mathrm{s}}\,, \quad w_0 = 4 \cdot 10^{-3}\,\mathrm{m}\,, \quad v = 0,1\,\frac{m}{s}\,, \quad \kappa = \frac{v\,w_0}{D} \approx 29 \quad . \tag{16}$$

The integrand is simple (see Fig. 3), it can be evaluated with a simple numerical integral formula (Simpson-formula). The integral was evaluated numerically with a C++ program by the Simpson-formula on an interval from 0 to 10^3 values divided by 10^6 equal parts. The following approximate formulas were fitted by the QtiPlot computer program, with a Levenberg-Marquardt algorithm:

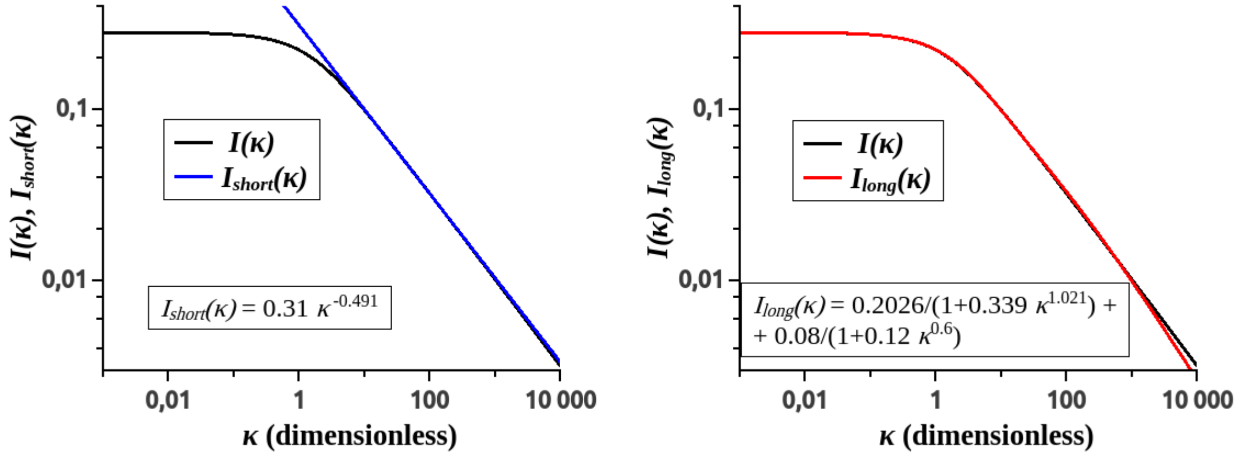
- If $10 < \kappa < 10^4$ then (see Fig. 4.a)

$$\Delta T_{max}\left(P, \lambda, w_0, \kappa\right) = \frac{P}{\lambda\,w_0}\,0.31\,\kappa^{-0.491} \quad , \tag{17}$$



a) b)

Figure 3. The integrand of Eq. (15) is a smooth function of s for a quite wide range of parameter κ



a) b)

Figure 4. The fitted formulas: (a) eq. (17), (b) eq. (18)

- If $10^{-3} < \kappa < 10^{3}$ then (see Fig. 4.b)

$$\Delta T_{max}\left(P, \lambda, w_0, \kappa\right) = \frac{P}{\lambda\, w_0}\left(\frac{0.2026}{1 + 0.339\,\kappa^{1.021}} + \frac{0.08}{1 + 0.12\,\kappa^{0.6}}\right) \quad , \tag{18}$$

where (just for a good local transparency)

$$\kappa = \frac{v\, w_0}{D} \quad , \tag{19}$$

v is the scanning speed, w_0 is the width of the beam, and $D = \frac{\lambda}{c\,\rho}$ is the diffusion constant, c is the specific heat capacity, and ρ is the density of the metal, $P = \tau\, P_0$ is the absorbed beam power, τ is the transmission constant of the metal, P_0 is the incident beam power.

Equation (17) can be used when the normalized scanning speed parameter has a much higher value then unity. This critical scanning speed, v_{cr} can be calculated with the use of eq. (19):

$$v_{cr} = \frac{D}{w_0} \quad . \tag{20}$$

For steal ($D = 13.7 \cdot 10^{-6}\, \frac{\mathrm{m}^2}{\mathrm{s}}$) it gives

$$v_{cr} \approx \frac{0.8}{w_0} \quad , \tag{21}$$

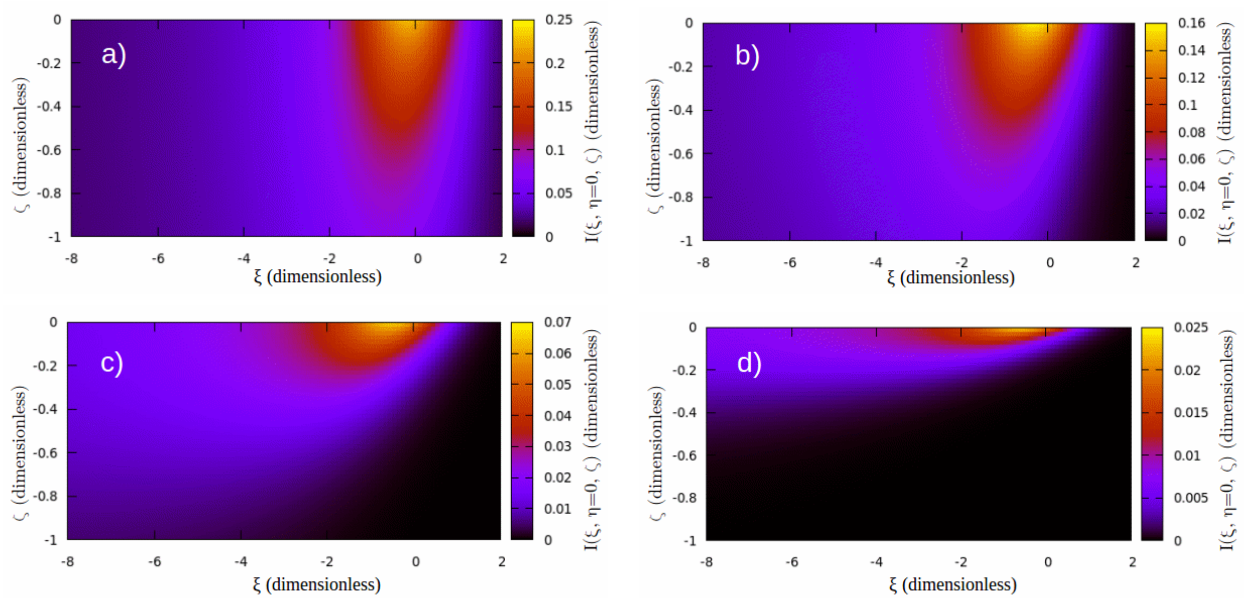


Figure 5. Temperature increase distributions at different dimensionless speed parameter, κ values, a: $\kappa = 0.5$, b: $\kappa = 4.0$; c: $\kappa = 32.0$, d: $\kappa = 256.0$

when the beam width parameter of the incident light beam, w_0 (see ref [6]) is given in millimeter then the value of the critical speed is obtained in meter per minutum.

Below this critical scanning speed the (15) integral gives approximately constant value, $I \approx 0.28$. For steal it gives (see eq. 14)

$$\Delta T_{max} \approx 6\,\frac{P}{w_0} \quad , \tag{22}$$

when the absorbed power, P is measured in watt, the beam width, w_0 is measured in mm then the temperature increase, ΔT_{max} is obtained in K or in C^{o}. It is important to remember, that this formula is valid only for metal blocks with dimensions much bigger than the characteristic spatial parameters of the problem.

6 Discussion

One can determine not only the temperature increase but also the whole temperature distribution from eq. (13):

$$\Delta T\left(\xi,\eta,\zeta\right) = \frac{P}{\lambda\, w_0}\, I\left(\xi,\eta,\zeta\right) \quad , \tag{23}$$

where the dimensionless $I\left(\xi,\eta,\zeta\right)$ integral is

$$I\left(\xi,\eta,\zeta\right) = \frac{1}{\pi^{\frac{3}{2}}} \int_0^\infty \frac{\mathrm{d}s}{(1+s^2)}\, \exp\left[-\frac{\left(\xi+\kappa\, s^2/4\right)^2+\eta^2}{1+s^2} - \frac{\zeta^2}{s^2}\right] \quad . \tag{24}$$

Figure 5. shows the I-integral dependence on ξ and η. It can be seen that as the scanning speed increased the temperature deviation is deceased, the temperature increase distribution is widened behind the light beam, and the heat effected depth is decreased.

It can be noticed that the maximal tempretaure increase appears not exactly at the center of the scanning light beam but some distance behind. Table 6. shows the values of the distance lags ($\Delta\xi$) and temperature differences (T_{max}) as a function of the dimensionless scanning speed parameter (κ) values.

It is important that the temperature deviation measured at the centre of the beam, T_0 and the maximal temperature deviation ΔT_{max} ($\Delta\xi$ behind the laser beam) are not differs more than 20%. That

κ	T(0)	dx	T-max	T-max/T(0)
0	0.280119	0	0.280119	1
0.5	0.247834	0.1	0.249166	1.005
1	0.222386	0.2	0.22572	1.015
2	0.18704	0.2	0.193926	1.037
4	0.146894	0.3	0.157534	1.072
8	0.109551	0.4	0.121775	1.112
16	0.079385	0.5	0.0906718	1.142
32	0.0567434	0.5	0.066031	1.164
64	0.0402964	0.5	0.0473799	1.176
128	0.0285192	0.5	0.0337267	1.183
256	0.0201407	0.5	0.0239024	1.187
512	0.0141986	0.5	0.0168944	1.19

Figure 6. The temperature increase is not maximal in the centre of the scanning laser beam, but it is delayed as the scanning is fast enough

is the approximation does not get wrong beacuse the temperature increase was taken as maximal at the centre of the scanning laser beam.

It is also interesting to notice, that the spatial lag of the position of the maximal temperature increase is stabilized to $\Delta\xi = 0.5$ if the dimensional scanning speed parameter, κ has much higher value than unity. The distances are normalized to the widthparameter of the laser beam (see eq. 11), that is the spatial lag of the maximal temperature increase is half of w_0 behind the centre of the laser beam if the scanning is fast enough (see table 6.).

7 Conclusion

The temperature increase caused by a scanning laser beam on a metal surface is investigated. New, simple formula is given for the maximal temperature increase as a function of the scanning speed, the width and the power of the laser beam, and the parameters of the metal (density, specific heat, heat conductivity, and absorptivity). The formula is simple, that is, it cannot be precise. It can be used in practice of laser-beam metal surface heating to guess the heating of the metal surfe at a given experimental conditions.

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